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14. ABSTRACT
We are studying adaptive social networks, focusing on spread of infectious disease as our primary example and including terrorist recruitment as an additional example. In an adaptive network, individuals change their social connections in response to their neighbors' characteristics, and these changes in network topology affect subsequent properties of the individuals. The network adaptation can be disease avoidance or connecting to potential recruits. Major goals of the project included extending previous models to incorporate more realistic network structure,

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Report Title

Adaptive Networks Foundations: Modeling, Dynamics, and Applications

ABSTRACT

We are studying adaptive social networks, focusing on spread of infectious disease as our primary example and including terrorist recruitment as an additional example. In an adaptive network, individuals change their social connections in response to their neighbors' characteristics, and these changes in network topology affect subsequent properties of the individuals. The network adaptation can be disease avoidance or connecting to potential recruits. Major goals of the project included extending previous models to incorporate more realistic network structure, adding spread of information that affects human behavior, studying the extinction of diseases, developing control strategies for epidemics on adaptive networks, and developing tools to analyze and monitor adaptive network properties. We have extended models to include network community structure, information spread, and more realistic social adaptation. We developed the first adaptive network model for terrorist recruitment. Our analytic work includes new techniques for predicting extinction rates of epidemics and the trajectory to extinction, methods to apply this to extinction on a network, and new moment closure approximation techniques that lead to more accurate predictions. For monitoring and control, we developed a method to quantify network adaptation and studied vaccine control for epidemics in adaptive networks.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
01/15/2013	8.00	Ilker Tunc, Maxim S. Shkarayev, Leah B. Shaw. Epidemics in Adaptive Social Networks with Temporary Link Deactivation, Journal of Statistical Physics, (12 2012): 0. doi: 10.1007/s10955-012-0667-7
08/31/2010	2.00	L.B. Shaw, I.B. Schwartz. Enhanced vaccine control of epidemics in adaptive networks (published in Physical Review E), Physical Review E, (04 2010): . doi:
08/31/2010	4.00	I.B. Schwartz, L.B. Shaw. Rewiring for adaptation (published in American Physical Society Trends in Physics), Physics, (02 2010): . doi:
08/31/2010	3.00	E. Forgoston, S. Bianco, L.B. Shaw, I.B. Schwartz. Maximal sensitive dependence and the optimal path to epidemic extinction (published in Bulletin of Mathematical Biology), Bulletin of Mathematical Biology, (03 2010): . doi:

TOTAL: 4

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received		Paper
08/26/2011	6.00	Eric Forgoston, Simone Bianco, Leah B. Shaw, Ira B. Schwartz. Converging towards the optimal path to extinction, Journal of the Royal Society Interface, (05 2011): 0. doi:
TOTAL:		**1**

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Invited presentations (given by Leah Shaw unless otherwise indicated):
• Epidemic spread in adaptive social networks: Effects of avoidance behavior. Complex Biological Systems Group Theme Days, Pittsburgh PA, May 4-5, 2012.
• Dynamics of infection spreading in an adaptive network with two coupled communities. AMS Fall Southeastern Section Meeting, Wake Forest University, Winston-Salem NC, Sep. 24-25, 2011. (Talk given by Ph.D. student and co-author Ilker Tunc.)
• Dynamics of epidemic extinction in adaptive social networks. SIAM Conference on Applications of Dynamical Systems, Snowbird UT, May 22-26, 2011.
• Dynamic convergence to the optimal path to stochastic extinction. Dynamics Days, Chapel Hill NC, Jan. 5-8, 2011.
• Epidemic spread and control in complex adaptive networks. Workshop on Complex Driven Systems: From Statistical Physics to the Life Sciences, Blacksburg VA, Oct. 1-3, 2010.
• Epidemic extinction in adaptive networks with random pulsed vaccination. SIAM Conference on the Life Sciences, Pittsburgh PA, Jul. 12-15, 2010.
• Social adaptation enhances vaccine control of epidemics in adaptive networks. USMA Network Science Workshop, West Point NY, Oct. 28-30, 2009.
• Controlling epidemic spread in adaptive networks using Poisson vaccination. SIAM Conference on Applications of Dynamical Systems, Snowbird UT, May 17-21, 2009.

Contributed presentations:
• Shkarayev, M. and Shaw, L.B. Asymptotically inspired moment-closure approximation for adaptive networks. American Physical Society March Meeting, Boston MA, Feb. 22-Mar. 2, 2012.
• Long, Y., Gross, T., and Shaw, L.B. Epidemic and information co-spreading in adaptive social networks. American Physical Society March Meeting, Boston MA, Feb. 22-Mar. 2, 2012.
• Shaw, L.B., Long, Y., and Gross, T. Simultaneous spread of infection and information in adaptive networks. Casablanca International Workshop on Mathematical Biology, Casablanca, Morocco, Jun. 20-24, 2011.
• Tunc, I. and Shaw, L.B. Dynamics of infection spreading in adaptive networks with communities. SIAM Conference on Applications of Dynamical Systems, Snowbird UT, May 22-26, 2011.
• Shkarayev, M.S., Shaw, L.B., and Schwartz, I.B. Recruitment dynamics in adaptive social networks. American Physical Society March Meeting, Dallas TX, Mar. 21-25, 2011.
• Shaw, L.B. and Tunc, I. Epidemic spread in adaptive social networks with community structure. BIONETICS, Boston MA, Dec. 1-3, 2010.
• Shaw, L.B. and Tunc, I. Epidemics in adaptive networks with community structure. American Physical Society March Meeting, Portland OR, Mar. 15-19, 2010.
• Shaw, L.B. Vaccination and social adaptation for epidemic control in networks. Boulder School for Condensed Matter and Materials Physics, Boulder CO, Jul. 6-24, 2009.
• Shaw, L.B. and Schwartz, I.B. Poisson vaccination for epidemic control in adaptive social networks. American Physical Society March Meeting, Pittsburgh PA, Mar. 16-20, 2009.

Poster presentations:
• Antwi, S.A. and Shaw, L.B. Stochastic dynamics in a social network with payoff-dependent adaptation. Dynamics Days 2012, Baltimore MD, Jan. 4-7, 2012.
• Long, Y., Gross, T., and Shaw, L.B. Interaction of epidemic and information spreading in adaptive networks. SIAM Conference on Applications of Dynamical Systems, Snowbird UT, May 22-26, 2011. • Tunc, I. and Shaw, L.B. Effects of community structure on epidemic spreading on adaptive networks. SIAM Conference on the Life Sciences, Pittsburgh PA, Jul. 12-15, 2010. (poster presentation by Tunc)

Number of Presentations: 20.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received		Paper
08/30/2011	7.00	Ira Schwartz, Leah Shaw, Maxim Shkarayev. Adaptive network Dynamics--Modeling and control of time-dependent social contacts, 14th International Conference on Information Fusion. 2011/07/05 00:00:00, . : ,

TOTAL: **1**

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
01/15/2013	9.00	Ira B. Schwartz, Maxim S. Shkarayev, Leah B. Shaw. Recruitment dynamics in adaptive social networks, Physical Review E (04 2012)
01/16/2013	10.00	Ilker Tunc, Leah B. Shaw. Effects of community structure on epidemic spread in an adaptive network, ArXiv e-prints (12 2012)
08/27/2009	1.00	Leah B. Shaw, Ira B. Schwartz. Enhanced vaccine control of epidemics in adaptive networks (submitted to Physical Review Letters), ( )
08/31/2010	5.00	I.B. Schwartz, E. Forgoston, S. Bianco, L.B. Shaw. Converging towards the optimal path to extinction (submitted to Journal of the Royal Society Interface), (08 2010)
TOTAL:		**4**

Number of Manuscripts:

Books

Received	Paper

TOTAL:

Patents Submitted

Patents Awarded

Awards

SIAM poster prize (awarded to top ~10% of posters) awarded to William and Mary Applied Science Ph.D. student Yunhan Long (advisee of Shaw, funded by this grant) for the following poster:

Long, Y., Gross, T., and Shaw, L.B. Interaction of epidemic and information spreading in adaptive networks. SIAM Conference on Applications of Dynamical Systems, Snowbird UT, May 22-26, 2011.

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Ilker Tunc	0.33	
Yunhan Long	0.33	
Shadrack Antwi	0.33	
FTE Equivalent:	**0.99**	
Total Number:	**3**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
Simone Bianco	0.26
Maxim Shkarayev	0.07
FTE Equivalent:	**0.33**
Total Number:	**2**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Leah Shaw	0.17	
FTE Equivalent:	**0.17**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:	0.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:......	0.00
The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:......	0.00
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):......	0.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:......	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:......	0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED

FTE Equivalent:

Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

See attachment.

Technology Transfer

Scientific progress and accomplishments

Leah B. Shaw

February 13, 2013

We are studying adaptive social networks, focusing on spread of infectious disease as our primary example and including terrorist recruitment as an additional example. In an adaptive network, individuals change their social connections in response to their neighbors' characteristics (e.g., infection status, susceptibility to radical views), and these changes in network topology affect subsequent properties of the individuals. For example, the network adaptation could be avoidance of a disease via rewiring of links between infected and noninfected individuals. Major goals of the project included extending previous models to incorporate more realistic network structure, adding spread of information that affects human behavior, studying the extinction of diseases, developing control strategies for epidemics on adaptive networks, and developing tools to analyze and monitor adaptive network properties. During the project, we have made progress in all of these areas. Results from the entire project period (February 2009 to February 2012) are summarized here.

1 New models for additional effects

We first discuss results of expanding or altering simpler adaptive network models for greater realism or to study different phenomena.

1.1 Network community structure

Previous models for epidemics in adaptive networks did not account for local community structures that occur in real social networks. Communities are groups of nodes that are closely interconnected to each other but weakly connected to the rest of the network. As the first model for community structure in an adaptive network, we studied a network with two communities. We adjusted the link rewiring rules so that the two communities were maintained despite the network evolution process. The two communities may be heterogeneous (e.g., different average connectivity in each), and we quantified the community structure by the number of cross links between the communities. We developed a stochastic computer simulation for the full adaptive network model with community structure and a lower dimensional mean field theory for dynamics of the nodes and links.

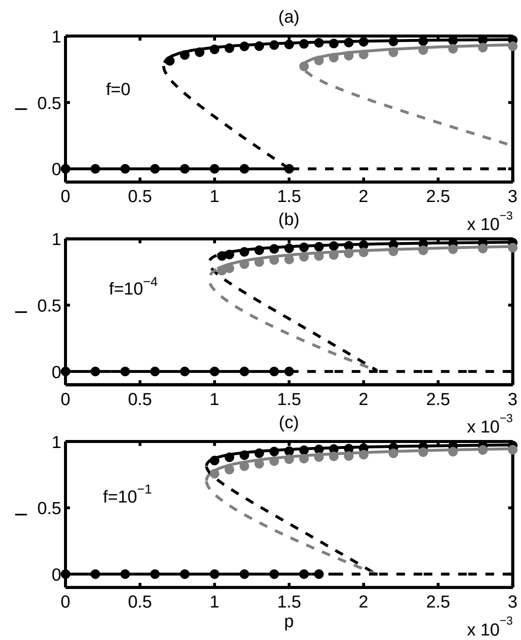


Figure 1: Steady state bifurcation diagram for infected fraction in an adaptive network with communities as a function of infection rate p for different fractions of cross links f between communities. Black: high connectivity community; gray: low connectivity community. Stable mean field solutions (solid curves) and simulations of the full system (circles) are in agreement. a) No cross links– community structure is absent. b) Few cross links (0.01% of the links). c) Moderate cross links (10% of the links).

We studied the dynamics of the two community system for various network geometries and parameter values. The mean field theory was in good agreement with the full model. We observed several important effects of community structure in adaptive networks that were not observed previously when this structure was absent. First, changes in community structure occurred in response to the epidemic spread, and these changes tended to have an equalizing effect on the connectivity within each community. Second, adaptation in the presence of community structure also had an equalizing effect on infection levels in the communities. As seen in Figure 1, even a small number of connections between communities (panel b) leads to similar infection levels in the two communities at steady state. This is a qualitatively different steady state than occurs in static networks, which can maintain low infection in a community that is weakly connected to a community with high infection. Equalizing of infection levels results from the adaptation increasing the number of connections in the low-connectivity community. A manuscript on this work has been posted to the preprint archive [17] and will soon be submitted to *Physical Review E.*

1.2 Information co-spreading in adaptive networks

Previous adaptive network models, including that discussed above, assume that all individuals are well informed and able to make beneficial decisions on how to change their social contacts. However, individuals may lack such information. We have modeled the simultaneous spread of both an epidemic and information *about* the epidemic. Information can spread through interpersonal contacts, appear spontaneously (e.g., individuals discovering the information on their own), or be obtained from an external source. Introducing better information into the system, for example by media campaigns, can serve as a source of control. There are several possible interpretations of the concept "information." Here we have chosen it to mean awareness of the need to change or rewire one's social contacts to prevent infection spread. We developed a computer simulation for an adaptive network with both epidemic and information spread, as well as a lower dimensional mean field theory for the system. Qualitative changes in the dynamics occur due to the presence of information, such as periodic oscillations in information and infection levels (Figure 2a). When only part of the population is informed, there are limits to the efficacy of avoidance rewiring in preventing disease spread. In particular, the epidemic threshold (transmission rate where spreading begins to occur) saturates at some rewiring rate, and increased efforts to avoid infection by rewiring do not further prevent an outbreak (Figure 2b).

We have also compared interpersonal communication and external media as sources of information. For a given fraction of informed individuals in the population, we compared whether communication or media information was more effective at suppressing disease transmission (Figure 3a). This depended on the infection level in the system. When infection is very prevalent, communication is more effective at reducing disease spread. Near the epidemic threshold, when infection levels are low, the reverse is true. We find that the threshold infection rate with media information is about twice that with communication (Figure

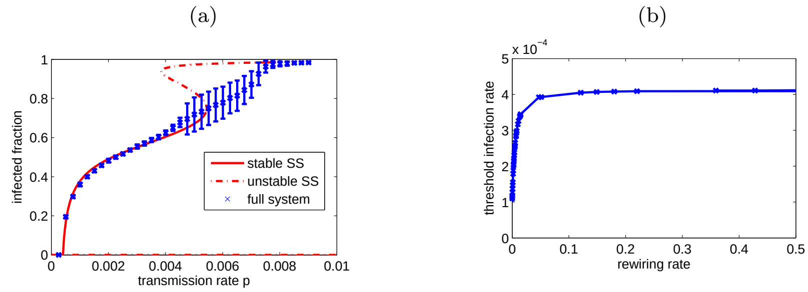


Figure 2: Epidemic in an adaptive network with partial information from media. (a) Bifurcation diagram for infected fraction versus infection rate. Curves are mean field steady states; symbols are simulations of the full system. Bars indicate standard deviations of time series. Large bars are associated with regions of periodic oscillations. (b) Threshold infection rate versus rewiring rate. When information is partial, the effect of avoidance rewiring is limited.

3b). Near the epidemic threshold, communication is less effective at suppressing epidemic spread because informed individuals break their links with infected neighbors, reducing the chances that those neighbors will learn the information. With external media, even an isolated individual has opportunities to become informed. Another difference between communication and media information is that when network adaptation is fast and information is communicated, the system can split into two disjoint groups: one with disease and one with information. People in the infected group are then isolated from the information and have limited opportunities to protect themselves. A manuscript on this work is in preparation [8].

1.3 Link deactivation

The studies discussed above, as well as many studies by other researchers, allow individuals to permanently rewire their social connections away from risky contacts. However, it is more realistic that people will temporarily avoid infected neighbors but resume contact when the risk is past. We have developed a new model in which links between susceptible and infected nodes can be deactivated and later reactivated once the infected node has recovered. The model is well described by our mean field theory, and we have analytically solved for the epidemic threshold and endemic infection levels. We find that bistability (stable endemic state coexisting with stable disease free state) does not occur with link deactivation, in contrast to models with link rewiring (e.g., Figure 1b,c).

We identify two parameter regimes in the system depending on the speed of the network adaptation. When the network changes slowly, adaptation reduces the number of active contacts per individual and thus reduces disease spread, but the details of which links are turned off are unimportant. The behavior in

(a) (b)

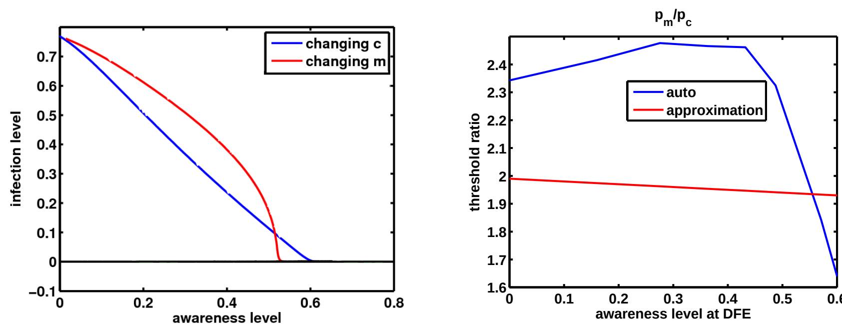
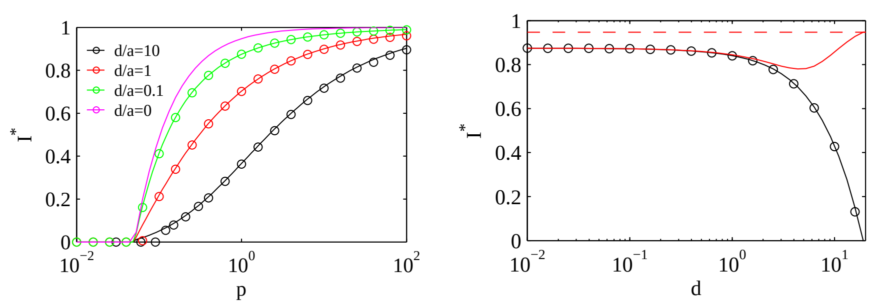


Figure 3: Comparison of media information and communication effectiveness at suppressing epidemic spread in an adaptive network. (a) Infected fraction versus informed fraction when information arises from communication (blue) or media (red). (b) Ratio of threshold infection rate with media information to threshold infection rate with communication, versus information level in population in the absence of disease. Blue: results of mean field model; red: results of a simple approximate argument.



(a) (b)

Figure 4: Network adaptation via temporary link deactivation. (a) Slow network dynamics: Infected fraction vs. infection rate for various ratios of link deactivation rate to reactivation rate (d/a). Symbols are simulation results for epidemic model on adaptive network with slow deactivation. Curves are predictions for static network with the same connectivity. (b) Fraction of infected nodes as a function of deactivation rate d. Symbols: simulation results for adaptive network with fixed ratio of deactivation and reactivation rates. The mean field solution for model with adaptation (black solid curve) is accurate over the whole parameter range. The static network with the same connectivity (red solid curve) matches only for slower network adaptation. The red dashed curve is for the original network with all links active.

this case is similar to epidemic spread on a static network with the same connectivity (Figure 4a). When the network dynamics are fast, adaptation removes dangerous connections in a targeted way and better controls the disease spread (Figure 4b). Interestingly, the connectivity of the network does not decrease monotonically as the link deactivation rate is increased. Instead, connectivity actually *increases* when deactivation becomes rapid enough to control the infection spread and the number of links needing to be deactivated decreases.

A paper on the link deactivation model has been published in *J. Statistical Physics* [18].

1.4 Risk/benefit-motivated decision making

As another approach to model how individuals may or may not perform self protective social adaptation, we are developing a model based on game theoretic ideas. Individuals change their connections based on an assessment of the risk and benefit of those connections. Motivated by sexually transmitted diseases, we assume that individuals derive a certain intrinsic benefit from making a connection with a neighbor (which may depend on the desirability of the neighbor). When we allow the network to adapt to the presence of a disease without immediately noticeable symptoms, individuals must assess risk based on the limited information they have, such as typical prevalence of the disease in the population. The perceived risk of a connection is assumed to increase as the infected fraction in the population increases. Further, perceived risk increases as the degree (number of connections) of the potential neighbor increases, thus increasing the likelihood that the potential neighbor has already been infected. We have developed a computer simulation of the system and a theoretical model to predict its behavior. Adaptive risk assessment reduces disease spread in the population when compared with a system that assumes constant risk (Figure 5a). The typical number of connections an individual makes (its average degree) emerges as a tradeoff between benefit and risk, which respectively cause creation and termination of links. The presence of infection leads to a net reduction in connections (Figure 5b). We are continuing to study the properties of the model as a function of parameters and will write a paper on this system for submission to a scientific journal.

1.5 Terrorist recruitment

As a non-epidemic example in adaptive social networks, we have modeled recruitment to a cause, such as terrorist recruitment, in adaptive networks. Many papers discuss terrorist networks as optimal structures that balance communication efficiency with maintaining the secrecy/security of the networks (e.g, [6]), while ignoring the dynamical processes that take place on such networks. Another class of work [1, 19] models the dynamics of terrorist recruitment within a well-mixed population, where the existence of network structure is excluded from the discussion, and therefore any network changes that may arise as a result of node dynamics are ignored. Models that include both terrorism recruitment

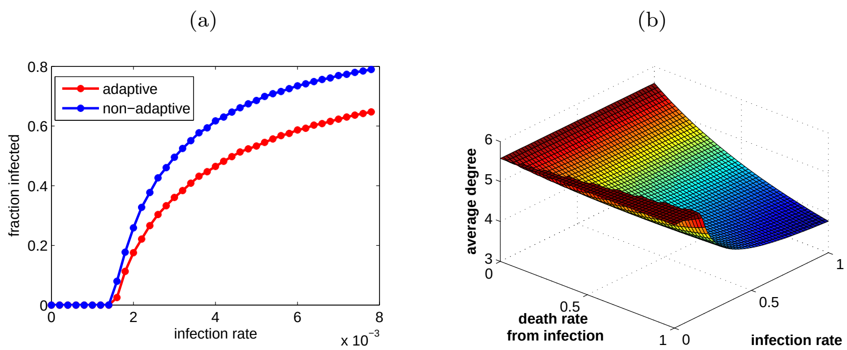


Figure 5: Risk-benefit system. (a) Fraction of population infected versus infection rate: comparison of system with adaptive risk assessment and system with no adaptation (fixed risk assessment). (b) Effect of infection on average degree (number of contacts) per individual in adaptive network. High infection rate is associated with large infected fraction and large reduction in number of contacts.

dynamics and network structure are extremely rare, and no such model has considered adaptive changes of the social network structure.

We have developed a model for the time evolution of a social network as terrorists recruit individuals from a pool that is susceptible to radical ideas. We modeled the network evolution with members belonging to one of three classes: non-susceptible, susceptible, and terrorist, following the categories used in [19] in the absence of network structure. By treating radical ideas as a phenomenon that can spread person-to-person along social contacts, we built on our previous studies of epidemic spread in networks. Terrorist nodes adapt by rewiring their connections away from non-susceptible nodes and toward susceptible nodes to maximize their recruiting ability. This type of rewiring is the opposite of the avoidance rewiring used in epidemic models, so the recruitment model provides an example of a new type of adaptation.

In addition to simulating the recruitment process on a finite population network, we have developed a mean field theory describing the system and have identified parameter regimes in which the mean field accurately predicts the terrorist level in the population (e.g., Figure 6). We have studied the interplay of the recruiting rate and the amount of adaptation. There are several parameter regimes of interest, depending on the fraction of individuals in the population that are susceptible to radical ideas, which will vary between countries. We have derived expressions for the threshold parameter values at which recruiting begins to be successful. We find that when susceptibles and terrorists are both at low levels, and individuals become susceptible to radical ideas infrequently, network adaptation is vital for successful recruiting. Preliminary results have been published in a conference proceedings [14]. More recent work has been

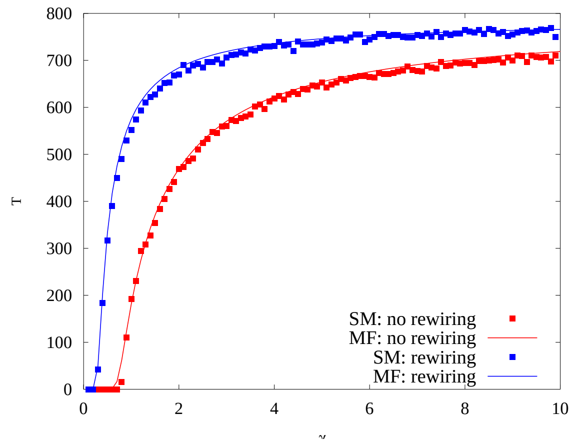


Figure 6: Dependence of number of terrorists on recruiting rate γ with (blue) and without (red) network adaptation. The simulations (points) and mean field theory (curves) are in good agreement.

posted on the preprint archive [16] and is under review at *Physical Review E*.

2 Analysis, monitoring, and control of adaptive networks

We next discuss our progress on methods for analysis, monitoring, and control of adaptive networks.

2.1 Epidemic extinction

It is desirable to understand the factors that promote epidemic extinction and to drive disease systems toward extinction/eradication. Prior to studying extinction in an adaptive network context, we have developed new analytic techniques in low dimensional globally coupled examples. In a stochastic system, small noise effects will drive the disease to extinction along an optimal (most probable) path. We have designed new tools based on variational principles to analyze this optimal path to extinction.

In a finite system of discrete individuals, extinction is inevitable in the long time limit unless there is a source of reintroduction. We begin with the master equation, which is a set of differential equations describing the time evolution of the probability to find the system in a given state at a particular time. Following [5], we make the assumption that extinction is a rare event whose probability decreases exponentially with population size. We maximize the extinction probability to seek the most probable path to extinction. This optimization problem

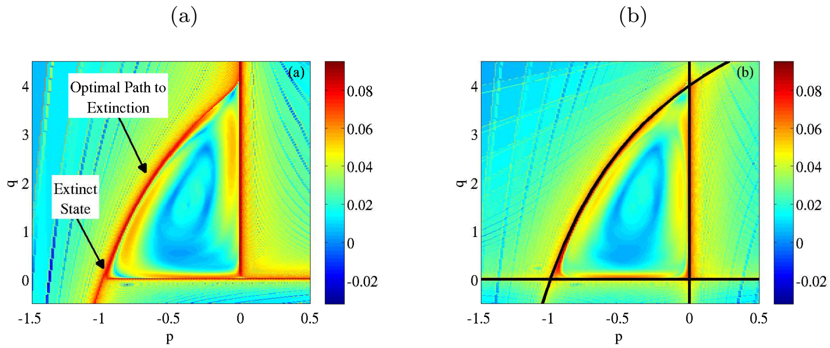


Figure 7: FTLE field (measure of sensitivity to initial conditions) for a single species population model. The vertical axis q is the population size (arbitrary units), and the horizontal axis p corresponds to a momentum due to noise. The optimal path connects the deterministic endemic state $(q \neq 0, p = 0)$ to the stochastic extinct state $(q = 0, p \neq 0)$ and represents the most probable path for the system to take to extinction. a) FTLE field. b) FTLE field with analytically determined optimal path overlaid.

for stochastic systems can be converted to a problem of finding a trajectory in a deterministic system with twice as many dimensions: position variables corresponding to the original population variables and momentum variables corresponding to the force due to the noise on each population variable. The optimal path is the trajectory carrying the system from the endemic state to a stochastic extinct state. Once it is found, we know the trajectory that the system is most likely to follow to extinction, and we can compute the extinction probability along the path to determine how the lifetime of the endemic state scales with parameters of interest.

We have shown that the optimal path has a saddle structure [2]. It forms a separatrix between two regions of phase space, and therefore the dynamics is very sensitive to initial conditions in the neighborhood of the optimal path. (For example, initial points on either side of the separatrix will diverge from each other in time.) We have developed a new method for locating the optimal path based on exploiting this sensitivity. We compute a field of finite time Lyapunov exponents (FTLE) [4], which are a measure of how quickly nearby trajectories diverge in time. We have shown that there is a ridge of maximal FTLE values that corresponds to the optimal path. Figure 7a shows the FTLE field for a single species population model. In Figure 7b, the analytically determined optimal path is overlaid and is observed to correspond to the ridge of maximal FTLE values. We have published two papers on this method for locating the optimal path to extinction [2, 13].

We have also developed a method for embedding lower dimensional models for which the optimal path is known (such as that of Figure 7) within higher

(a)   (b)

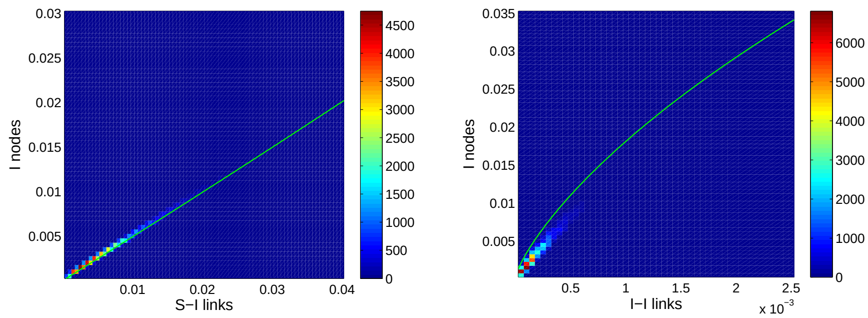


Figure 8: Most probable path to extinction for a SIS (susceptible-infected-susceptible) epidemic model on a static social network. Colors indicate the probability density function averaged over multiple stochastic trajectories to extinction. The predicted optimal path to extinction from our theory is overlaid (green curve). Two 2-dimensional projections of the same optimal path are shown for visualization purposes. (a) Projection onto subspace of I nodes vs. SI links. (b) Projection onto subspace of I nodes vs. II links.

dimensional models of interest. Numerically locating the optimal path can be difficult in high dimensional systems, but this embedding allows us to track the optimal path by starting with the known path and continuously varying a parameter. We have derived approximate dynamical equations for epidemic spread on a static social network, which describe the time evolution of the nodes and the links, and we have embedded a simpler epidemic model (without a network structure) within this higher dimensional system. Our continuation approach allows us to determine the most probable path to epidemic extinction within a social network. Sample results are shown in Figure 8. Our predicted optimal path is in good agreement with trajectories observed in stochastic simulations that ended in extinction. Although tracking an optimal path as a parameter slowly varies is easier than finding a path without prior information, this remains a complicated, high dimensional problem. Recent work by Lindley and Schwartz has led to improved numerical methods for finding optimal paths [7], and we are working with these researchers to combine our approaches. These ideas will be extended to extinction in adaptive networks, including extinction in the presence of vaccine control strategies [15]. We have observed previously that vaccination and adaptation work in synergy to reduce the epidemic lifetime by orders of magnitude.

2.2 Improved moment closure for better mean field theories

The mean field theories we have discussed are equations for the time evolution of nodes and links. However, changes in the number of links of each type depend on higher order network structures, such as triples of nodes. Generating a mean field theory requires closing the system of equations by approximating higher order structures in terms of nodes and links. The standard approach to moment closure [3] is based on a homogeneity assumption. The neighborhood of each node is assumed to be similar to that of every other node of the same type. Although this assumption often works well for simple models without much long range correlation, we have found parameter regimes for the terrorist recruitment model in which the mean field theory is less accurate due to approximations about the network geometry. Inaccuracies have also been identified in certain parameter regimes in adaptive epidemic spreading models [9]. Developing more accurate mean field theories is desirable because the mean field system (~ 10 equations) is so much lower dimensional than the full network system (usually $\sim 10^4$ nodes in our simulations) or even heterogeneous mean field theories developed by others (e.g., [10]) ($\sim 10^2$ equations but becomes much larger if nodes can have several states).

We have introduced a new approach to moment closure to generate more accurate low dimensional mean field models. Rather than making an ad hoc homogeneous assumption for how triples of nodes depend on link and node variables, we obtain a different closure by calculating how the triples depend on nodes and links in the limit of extreme parameter values, such as rapid network adaptation. We then bring this asymptotically derived closure to standard parameter ranges, where we find that it can still hold. Figure 9 shows some of our improved mean field results using the new moment closures for both the terrorist recruitment model and an epidemic spreading model. A paper about the moment closure techniques is in preparation.

2.3 Monitoring

We have developed a metric to quantify adaptive network structures. In an adaptive network, the network structure and the status of the nodes interact in a feedback loop. We used mutual information [12] between time-shifted time series to look for evidence of feedback behavior in our adaptive epidemic spreading model. Peaks were expected in the mutual information as a function of time shift, which would indicate that the node status at earlier times is predictive of the network geometry at later times and vice versa. Figure 10 shows the network geometry, measured by the degree (number of neighbors) of a particular node, responding to the node's status. However, the response of the node's status to the network geometry was not clear in our system, probably because the feedback in this direction is weaker. This metric will be applied to a more suitable test case in the future.

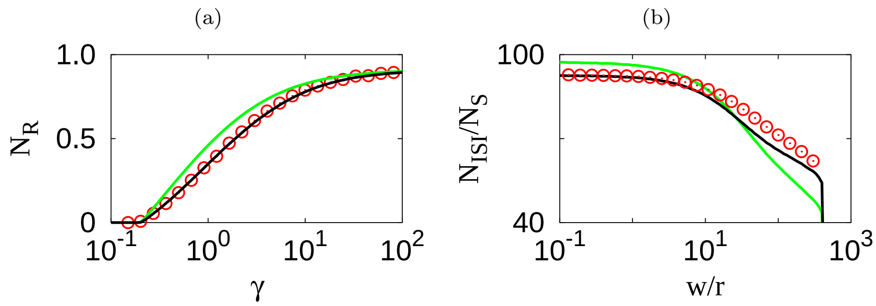


Figure 9: Results of improved moment closures for adaptive networks. Green curve: prediction from standard approximation; black curve: prediction from new moment closure; red circles: observed values from simulation. (a) In adaptive recruitment model, recruited fraction vs. recruitment rate. (b) In epidemic spread model, number of infected-susceptible-infected node triples per susceptible node, vs. link rewiring rate.

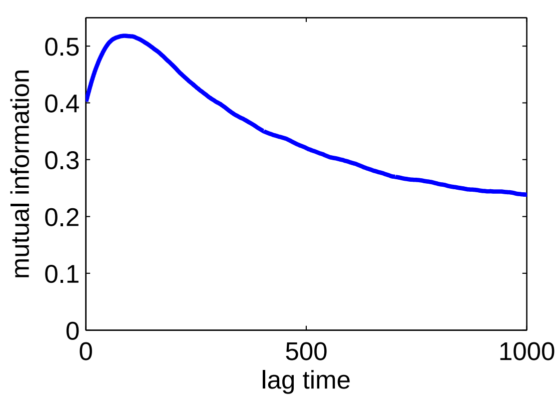


Figure 10: Mutual information for network structure responding to node status versus lag time. The structure responds to the node in approximately 100 time units.

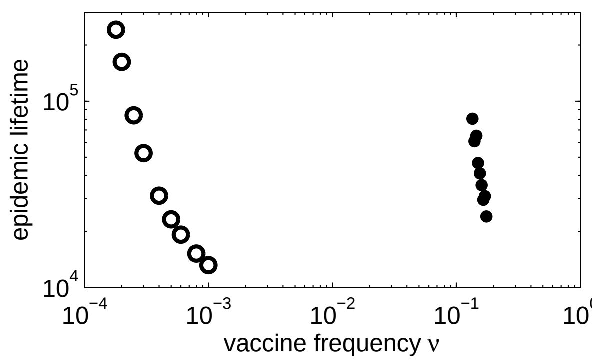


Figure 11: Dependence of endemic state average lifetime on vaccine frequency ν. Closed circles are for a static network and open circles an adaptive network. Two orders of magnitude less vaccination is needed to drive the epidemic to extinction in the adaptive network case.

2.4 Control

We have also studied control strategies for epidemics in adaptive networks. Control was implemented by adding random pulsed vaccination of susceptibles. We showed that vaccine control is much more effective in adaptive networks than in static networks due to an interaction between the adaptive network rewiring and the vaccine application. Orders of magnitude less vaccine application was needed to drive the disease to extinction in an adaptive network than in a static one (Figure 11). The reason for the synergy between network adaptation and vaccination is the following. The social adaptation leads to susceptible nodes having higher than average degree, because individuals are trying to reduce their connections to infected nodes. When vaccination of susceptibles occurs, this automatically selects the highest degree nodes to be vaccinated. Targeting of high degree nodes is known to be an effective vaccination strategy in static networks [11], and in adaptive networks this process happens automatically without requiring knowledge of the whole network geometry or targeting of specific nodes. This work has been published in *Physical Review E* [15].

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